10 February 2020

Review 1

This review is meant as a general overview of SOME of the topics covered in class up to date. The test questions will not only cover this material but will also cover sections 1.1-1.4 & 2.1-2.7, and implicitly all material from precalculus. You should know all definitions, theorems and techniques outlined in the text, and be comfortable with the properties and examples throughout the above sections as well as know how to solve the exercises and homework problems. Below I provide some sample problems that cover material from class. I am in no way promising any of these problems will be on the test. They are solely for practice. Other good sample problems can be found in your homework assignments, lecture notes, and class handouts, as well as the before mentioned places. Other great problems are found in the Review Exercises at the end of chapters 1 & 2.

(1) What are the domain and ranges of the six inverse trig functions? What do their graphs look like?
(2) What old functions have been reviewed in these chapters? What new functions have been defined in these chapters? How do all of these functions act and what do their graphs look like?
(3) What are the transformations one can perform on a standard graph? If $y = f(x)$ is a transformation of a standard function, what form for $f(x)$ is most useful in understanding the transformations?
(4) Solve the below equations for x.
   (a) $log_5 3x = 75$
   (b) $7^{2x-8} = 49$
   (c) $\cos 3x = \pi/6$
   (d) $|x - 7| = 14$
(5) Consider the position function $s(t) = -100t^2 + 250t$ where t is given in seconds and $s(t)$ is in meters.
   (a) Create a chart that displays the average velocity of the object over the intervals: $(2, 3)$, $(2, 2.5)$, $(2, 2.1)$, $(2, 2.01)$, $(2, 2.001)$.
   (b) Make a conjecture about the instantaneous velocity at $t = 2$ seconds.
(6) Compute the following limits.
   (a) $\lim_{x\to 1} \frac{1-x}{1-\sqrt{x}}$
   (b) $\lim_{x\to 3} \frac{x^2-9}{x-3}$
   (c) $\lim_{x\to 0} \frac{2x+4}{5x-9}$
   (d) $\lim_{x\to 0} \frac{\tan 2x}{x}$
   (e) $\lim_{x\to 0^+} x - \ln x$
(7) Consider the piecewise function:

$$f(x) = \begin{cases} 3 & x \le -3 \\ x & -3 < x < 0 \\ 1 & x = 0 \\ x^2 & 0 < x < 3 \\ 3 & x \ge 3. \end{cases}$$

   (a) Graph the function $f(x)$.
   (b) Determine all input values where the limit exists, where only a one-sided limit exists, and where $f(x)$ is continuous.
   (c) Evaluate all of the one-sided limits at the values above.
   (d) Are any of the discontinuities removable?
(8) Determine a continuous extension $H(x)$ at $x = 4$ for the function:

$$h(x) = \frac{(x^2 - 1)(x - 4)}{x^2 - 3x - 4}.$$

(9) State the Intermediate Value Theorem. How is this theorem used to show that certain polynomials have roots over specific intervals? Does the function $k(x) = \theta^3 + 4\theta + 9$ definitely have a zero between -2 and 0? What about between -1 and 0?

(10) Determine all asymptotes of the functions below.

$$s(x) = \frac{3x^3 - 6x^2 + 3x - 9}{x^2 + 1}$$

(11) Consider the function $t(x) = \frac{x}{x-5}$.
   (a) What is the average rate of change of $t(x)$ over [0,3]?
   (b) Determine the equation of the secant line to $t(x)$ connecting points with x-values 0 and 3 and write it in slope-intercept form.
   (c) What is the slope of the tangent line at $x = 3$?
   (d) Determine the equation of the tangent line to $t(x)$ at the point with x-value 3 and write it in slope-intercept form.

(12) Given that $-|x| \leq \sin x \leq |x|$, prove that $\lim_{x\to 0} \sin x = 0$.